Running at Altitude: the 100-metre Dash

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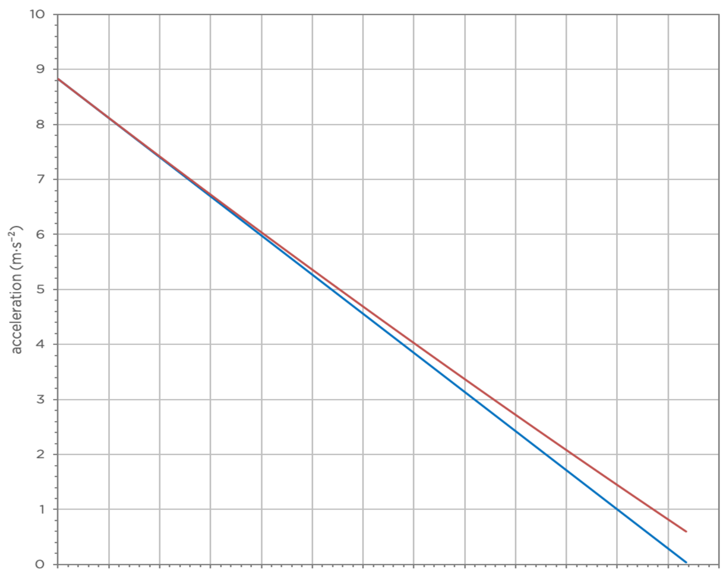
Theoretical 100 m performance times of a top athlete at Mexico City, Alto Irpavi (Bolivia) (2,250 and 3,340 m a.s.l.) and in a science fiction scenario "in vacuo" are estimated as follows. At the onset of the run: i) the velocity (v) increases exponentially with time; hence ii) the forward acceleration (a_f) decreases linearly with v, iii) its time constant (τ) being the ratio between v_{max} (for $a_f = 0$) and a_{fmax} (for v = 0). The overall forward force per unit of mass (F_{tot}), as given by the sum of a_f and of the air resistance ($F_a = k \cdot v^2$, where the constant $k \approx 0.0037$ $J \cdot s^2 \cdot kg^{-1} \cdot m^{-3}$) was obtained from the actual relationship between the instantaneous values of a_f and v during Usain Bolt's extant world record (see Figure). Since the decrease of k at altitude (due to the reduced barometric pressure, and hence air density) is known, assuming that F_{tot} is unchanged, the relationships between a_f and v at the altitudes in question were obtained subtracting the appropriate F_a values from F_{tot}. The 100 m performance times, as obtained from the resulting v_{max} and τ values in the three conditions considered amounted to 9.515, 9.474 and 9.114 s, as compared to 9.612 s at sea level. Performance times were also estimated from the relationship between overall mechanical power and speed, assuming that at the end of the run (when $a_f = 0$, and $v = v_{max}$) the mechanical power is unchanged regardless of altitude, thus leading to increased v_{max} values because of the reduced power dissipated against the air resistance. Since a_{fmax} (for v = 0), which is obviously independent of altitude, is know (see Figure), the so obtained v_{max} values, allowed us to estimate the appropriate time constants (τ) and the corresponding performance times which amounted to 9.474, 9.410 and 8.981 s. In conclusion, the rather small difference (≈ 0.3 %) between the estimated 100 m time (9.61 s) at sea level and the actual 100 m world record by Usain Bolt (9.58 s) supports the validity of the above approach. Hence, neglecting science fiction scenarios, the 100 m performances times of a top athlete at the two altitudes considered can be reasonably expected to be shorter by 1.0 to 1.4 % at Mexico City and 1.4 to 2.1 % at Alto Irpavi, the slight differences (0.43 to 1.46 %) between the two set data obtained from the two different estimates of τ reported above being likely due to the uncertainties in the underlying estimates of k and/or of the mechanical power.



Forward acceleration (a_f, $m \cdot s^{-2}$, blue) and overall forward force per unit body mass (F_{tot}, $N \cdot kg^{-1}$, red) as a function of the speed ($m \cdot s^{-1}$) during Usain Bolt's world record performance.